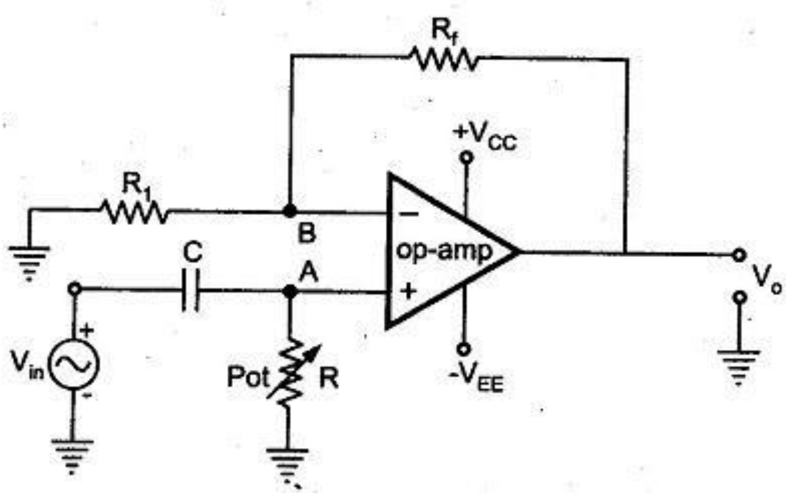
First Order High Pass Butterworth Filter

A high pass filter is a circuit that attenuates all the signals below a specified cut off frequency denoted as f_L. Thus, a high pass filter performs the opposite function to that of low pass filter. Hence, the First Order High Pass Butterworth Filter circuit can be obtained by interchanging frequency determining resistances and capacitors in low pass filter circuit. The first order high pass filter can be obtained by interchanging the elements R and C in a first order low pass filter circuit.



The frequency at which the gain is 0.707 times the gain of filter in pass band is called as low cut off frequency, and denoted as f_L. So, all the frequencies greater than f_L are allowed to pass but the maximum frequency which is allowed to pass is determined by the closed loop bandwidth of the op—amp used.

Analysis of the Filter Circuit

The impedance of the capacitor is

$$-jX_C = -j\left(\frac{1}{2\pi f C}\right)$$

where f is the input i.e. operating frequency. By the voltage divider rule, the potential of the non inverting terminal of the op—amp is

$$V_A = V_{in}\left[\frac{R}{R - j X_C}\right] \quad \ldots (1)$$

$$V_A = V_{in}\left[\frac{R}{-j X_C\left(\frac{R}{-j X_C} + 1\right)}\right] \quad \text{taking } - j X_C \text{ outside}$$

$-\frac{1}{j} = j$, we can write,

$$\frac{1}{-j X_C} = \frac{j}{X_C} = \frac{j}{\left(\frac{1}{2\pi f C}\right)}$$

$$= j\, 2\pi f C \quad \ldots (2)$$

Substituting in the above expression of VA,

$$V_A = V_{in}\left[\frac{\left(-\frac{R}{j X_C}\right)}{\left(-\frac{R}{j X_C}\right) + 1}\right]$$

$$V_A = V_{in}\left[\frac{j\, 2\pi f R C}{1 + j\, 2\pi f R C}\right] \quad \ldots (3)$$

This can be represented as

$$\therefore \quad V_A = V_{in}\left[\frac{j\left(\frac{f}{f_L}\right)}{1 + j\left(\frac{f}{f_L}\right)}\right]$$

where $f_L = \frac{1}{2\pi R C}$ = low cut off frequency ... (4)

Now, for the op-amp in non-inverting configuration,

$$V_o = A_F V_A$$

where V_A = Voltage at the non inverting input

and $A_F = \left(1 + \frac{R_f}{R_1}\right)$ = gain of op–amp in pass band

$$\therefore \quad V_o = A_F V_{in} \left[\frac{j\left(\frac{f}{f_L}\right)}{1 + j\left(\frac{f}{f_L}\right)}\right]$$

$$\frac{V_o}{V_{in}} = A_F \left[\frac{j\left(\frac{f}{f_L}\right)}{1 + j\left(\frac{f}{f_L}\right)}\right] \qquad \ldots (5)$$

This is the required expression for the transfer function of the filter. For the frequency response, we require the magnitude of the transfer function which is given by,

$$\left|\frac{V_0}{V_{in}}\right| = \frac{A_F\left(\frac{f}{f_L}\right)}{\sqrt{1 + \left(\frac{f}{f_L}\right)^2}} \qquad \ldots (6)$$

The equation (6) describes the behaviour of the high pass filter.

1) At low frequencies, i.e. $f < f_L$

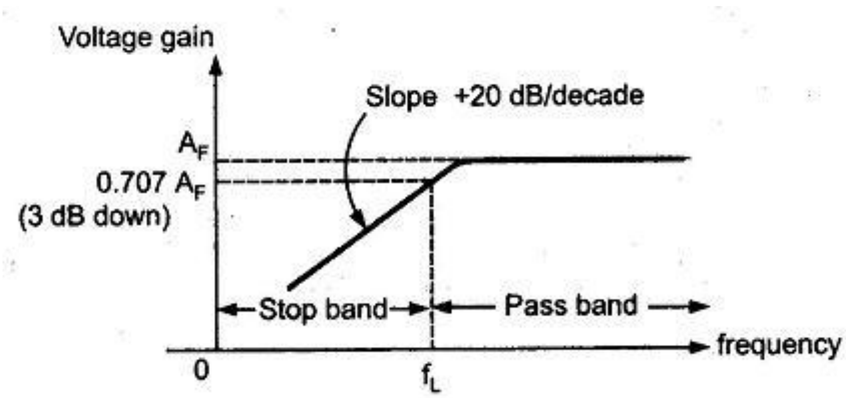
$$\left|\frac{V_0}{V_{in}}\right| < A_F$$

2) At $f = f_L$,

$$\left|\frac{V_0}{V_{in}}\right| = 0.707\ A_F \text{ i.e. 3 dB down from the level of } A_F$$

3) At $f > f_L$, i.e. high frequencies, 1 can be neglected as compared to $\left(\frac{f}{f_L}\right)$ from denominator.

$$\left|\frac{V_0}{V_{in}}\right| \cong A_F \text{ i.e. constant}$$



Thus, the circuit acts as high pass filter with a passband gain as A_f. For the frequencies, f <f_L, the gain increases till f = f_L at a rate of + 20 dB/decade. Hence, the slope of the frequency response in stop band is + 20 dB/decade for first order high pass filter. The frequency response is shown in the Fig. above.